Response characterisation in CFRP notched coupons with energy-based multi-axial failure data

Andrew J. Litchfield[1]*, Rodney S. Thomson[2], Adrian C. Orifici[1]

[1] *School of Aerospace, Mechanical and Manufacturing Engineering, RMIT University, GPO Box 2476, Melbourne, Victoria 3001, Australia*
** corresponding author: a.litchfield@student.rmit.edu.au, phone: +61 3 9925 6215, fax: +61 3 9925 6108*
[2] *Cooperative Research Centre for Advanced Composite Structures Limited 506 Lorimer Street, Fishermans Bend, Victoria, 3207, Australia*

Abstract

A methodology that uses multi-axial testing and dissipated energy (DE) to characterise the mechanical behaviour of laminated composite materials has been further extended to make use of intuitive relationships between material stiffness and DE. The methodology is able to accurately predict the initiation and early progression of damage providing valuable information on the location, intensity and behaviour of damage in a composite specimen. A constitutive relationship was implemented in which the material stiffness was degraded based upon the total work and dissipated energy. The resultant material behaviour demonstrated that response characterisation of composites with complex failure modes using DE is possible, even when employing a simplistic relationship. This suggests a more detailed constitutive relationship will lead to a more accurate prediction of load response and damage behaviour of composite materials.

Keywords: CFRP composites, damage modelling, multi-axial material characterisation

Introduction

Current aircraft structural design utilising fibre-reinforced polymer (FRP) composite materials is yet to fully exploit their capabilities due to the difficulties in capturing material behaviour up to and including failure. The present design and certification of aircraft composite structures is based on gathering and correlating experimental data from limited single axis tests and extrapolating this material data to actual structures. This becomes difficult and unsafe if trying to extrapolate the data to conditions and configurations outside the range of the tests. Furthermore, despite a plethora of composite failure criteria [1], most are yet to provide a satisfactory degree of predictive capability, such as what is required in demanding applications like aircraft primary structures. Both of these aspects lead to the requirement for tedious and expensive experimental testing at all critical length scales. This building block approach has arisen due to the inability of current composite failure theories to fully describe structural performance across length scales. In response, an approach has been implemented based on characterising the material behaviour in the complete loading space through multi-axial testing [2-4]. This has the potential to increase reliability and reduce the time and cost of the design and validation cycle by removing tiers of the building block approach, and may also allow the safe operation of composite structures with reduced conservatism.

In the current paper, the multi-axial characterisation approach has been implemented into a commercial finite element (FE) package with the aid of a custom user subroutine. This approach involves determining the damage function of a material from a sequence of tests covering the complete loading space. From this testing the “dissipated energy density function” or DED function can be characterised. The volume integral of this DED function equals the energy dissipated during loading due to the various internal failure events within the material. This mechanical strain-based DED function, fully characterised from experimental data, should be able to accurately describe material behaviour in terms of dissipated energy (DE) due to energy absorbing damage mechanisms, through the linear and non-linear regimes. This captures the collective behaviour of the damage mechanisms without needing to know the precise damage events. The DED function can be related to local stiffness changes and be used to model non-linear material behaviour.

A multi-degree of freedom (DOF) experimental testing regime is currently being pursued using unidirectional carbon fibre-reinforced polymer (CFRP) epoxy tape specimens and a novel hexapod testing machine at the U.S. Naval Research Laboratory. Further development and refinement of the approach has been performed by using the composite material damage model within Abaqus to produce “synthetic” characterisation data. This synthetic data has been found to be particularly useful in troubleshooting and preparing the methodology for the introduction of experimental multi-axial loading data, including out-of-plane deformation. This paper focuses on the utilisation of the synthetic data to assess the development of damage in double-notched and open-hole CFRP specimens under 3-DOF loading, with preliminary integration of a constitutive material law into the approach.

Generation of Synthetic Data

A detailed discussion on synthetic data was given by Orifici et al. [5], and only a brief outline is provided here. Firstly, the loading cases of interest are created as FE input files. For the work described in this paper, models were created in the commercial FE package Abaqus [7]. Modelling the configuration of the characterisation specimens, as shown in Fig. 1 was achieved using a single layer of quadrilateral shell elements. To capture dissipated energy, the damage model for fibre-reinforced composites in Abaqus was employed, and the model solved using Abaqus/Explicit [7]. The damage model uses the Hashin criteria to capture the initiation and progression of four types of composite-specific failure modes including: fibre rupture in tension; fibre buckling and kinking in compression; matrix cracking under transverse tension and shearing; and matrix crushing under transverse compression and shearing. These damage modes are used to trigger a progressive loss in stiffness. During this process, Abaqus calculates the energy associated with all damage modes, ALLDMD [7].

The FE results on their own however are not enough, as the goal is to replicate experimental output with the same data and layout as given by the test machine. To do this, a custom Python script [6] retrieves the data of interest such as DE and strains from the Abaqus results database and constructs a synthetic data test file, which is in the same format as actual experimental data files. Once all these data files have been prepared, material characterisation using the DED approach can begin.

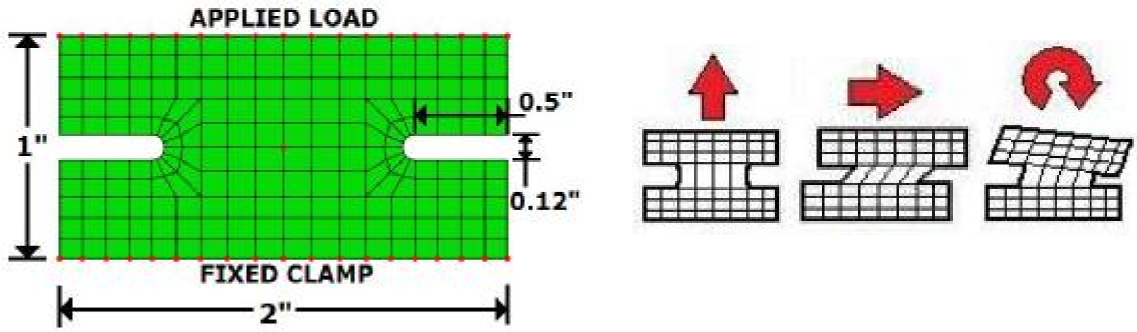

Fig. 1 Left: Geometry and dimensions of the double-notch characterisation specimen. Right: Fundamental pure load cases

Material Characterisation Constants

It has been postulated [2] that there exists a scalar function ϕ that expresses a measure of the dissipated energy density per unit of volume of material, and only depends on the strains and the material used in the structure. Determining the DED function for a material requires knowing the total dissipated energy experienced as well as the associated full-field strain distribution. The dissipated energy is the result of considering the total energy imparted to the specimen and the total recoverable energy after elastic unloading – the instantaneous difference between the two is the current amount of dissipated energy. Strain measurement is achieved using full-field optical strain measurement systems, and in the case of synthetic data, the full-field strains are taken from the FE analysis.

An example of a typical history plot of dissipated energy for an analysis with the Abaqus damage model is shown in Fig. 2. Points of interest include the initial period of zero DE corresponding to the elastic loading regime of the material, then at approximately 0.3×10^{-3} s when the material starts to soften and energy is dissipated. By approximately 0.6×10^{-3} s even though the material may still be straining, energy is no longer being dissipated.

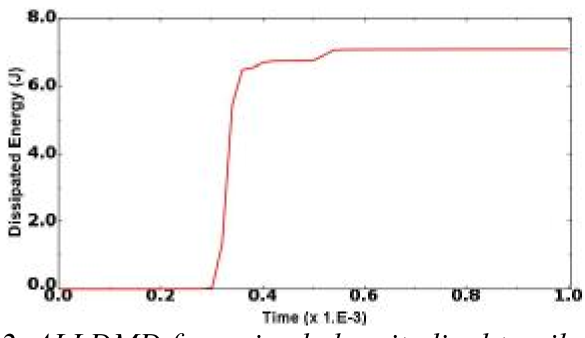

Fig. 2. ALLDMD for a simple longitudinal tensile case.

A material's unique DED characterisation constants are determined using the procedure explained in Ref. [1], written into Matlab©. The DED function can take any form expressed in terms of a set of unknown DED coefficients and known constant basis functions. In the interests of simplicity a linear polynomial form was chosen:

$$\phi = \int_{S_0} \left(c_1 \chi_1 \left(\varepsilon^P \right) V^P + \ldots + c_i \chi_i \left(\varepsilon^P \right) V^P \right) dx \tag{1}$$

The variables c_i represents the material dependent characterisation constants and χ_i the basis functions depending only on strains. Both are defined for i distinct points in a strain space. The integral of the function over the structure, S_0, gives the total dissipated energy. The concept of a strain space is explained in detail in Ref. [1].

The general procedure used to derive the characterisation constants from experimental or synthetic test data is as follows:

1. Gather test strain and DE data for the material being characterised.
2. Process strains to locate their positions within the strain space and construct the basis function matrix.
3. Solve for the vector of unknown characterisation constants by minimising the error between the DE determined analytically and that from test data.

Processing the strains involves taking their value at discrete points over the surface of a structure, for individual increments of a loading case and "locating" their position within the strain space. This is done by identifying whether the strain at a particular point is within certain ranges along each axis of the strain space, whereupon it is then said to reside inside a "strain space element". With its location identified, the strain is used to determine a normalised weighting based on the strains at chosen "known" locations at the boundaries of the element, i.e. at the eight corners of the cube-shaped strain space element (for 3-DOF). From Equation (1), each strain is also multiplied by the volume of the node (location) from whence it came. This process is repeated at all strain locations over the structure and for all increments in the loading regime. Repeating this process for a number of tests and multiple layup configurations builds a large matrix of basis functions, where the values are the result of the located and interpolated strain coordinates multiplied by the volumes of material at the strain locations.

From Equation (1), multiplying the basis function by a set of material-dependent coefficients gives the DE. The coefficients, c_i, are chosen such that the basis function multiplied by the coefficients plus some error, e, will give the experimental dissipated energy, as shown in Equation (2), where n is the number of discrete locations the strain is sampled from, p is the number of loading increments and i is the coefficient number.

$$\sum_{1}^{p} \sum_{1}^{n} c_i \chi_i \left(\varepsilon_n^p \right) V_n^p + e^p = DE^p \tag{2}$$

The large amount of data contained within the basis function equations and dissipated energies leads to a highly over-determined system with no unique solution for c_i. Minimising the norm of the error vector e^p provides the best approximation to the solution. In Matlab© a constrained linear least-squares curve fitting function was used to calculate the characterisation constants. Since the constants represent the dissipated energy density at known locations within the discretised strain space, they must be positive. To enforce this, the numerical optimisation is bounded by a minimum of zero.

Multi-sequence layup characterisation

As the physical experimental tests cover four non-symmetrical layup configurations ±15°, ±30°, ±60° and ±75°, their synthetic counterparts provide convenient baseline data for individual and group characterisation of the DED function. All four configurations were analysed, and material characterisation was performed using individual configurations and repeated using all the configurations in a combined data set. A more detailed discussion on the characterisation of these single and combined configurations is given in Litchfield et al. [8].

Grouping the four layup configurations together with ten unique loadcases each and fifty increments per loadcase, gave 2000 equations. Using this large number of equations, the maximum discretisation was applied to the strain space in the form of 11×11×11 segments along each strain direction creating 1331 strain space segments. The 1728 nodes at the boundaries of these segments were the unknown coefficients of DED. Minimising the error in Equation (2) gave the DED coefficients as shown in Fig. 3. In this Figure, the large number of non-zero coefficients indicates that the strains have been successfully weighted over a number of the coefficients, thus providing a higher resolution DED function when compared with past characterisations [8].

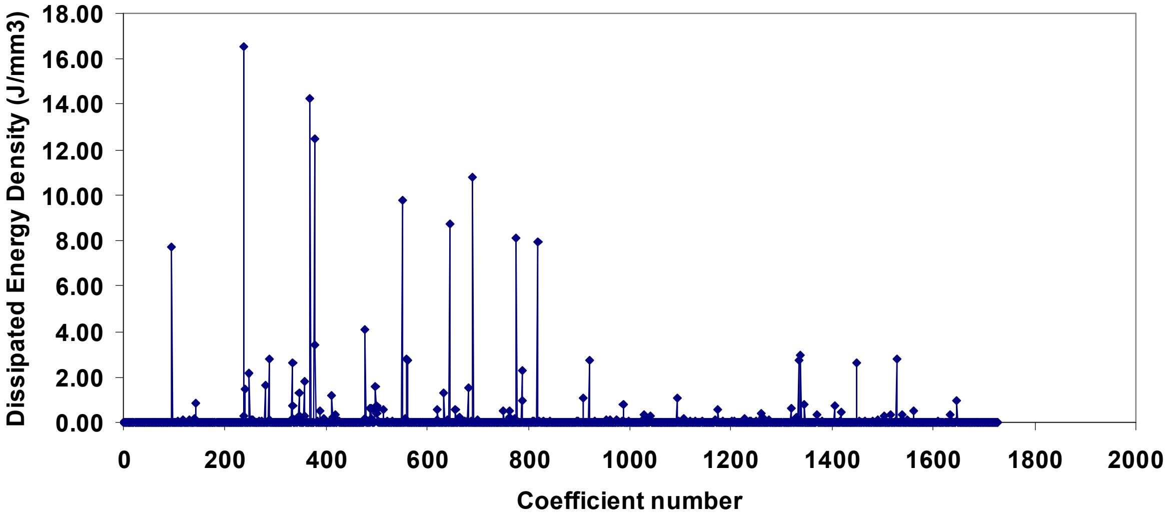

Fig. 3. Example of characterisation constants from characterisation of the material using all loadcases and layups

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A greater number of coefficients and higher strain space discretisation implies reduced error from the linear interpolation, and also allows the contribution of a strain path to the total dissipated energy to be weighted towards several sets of strain space nodes rather than just one or two sets. Intuitively, and as was observed during the characterisation, the strains from all loadcases originated from the centre of the strain space or where $\varepsilon_{11} = \varepsilon_{22} = \varepsilon_{12} = 0$. This meant that the central element/s of the strain space had an inordinate number of strains weighted towards their coefficients. This had the unexpected effect of artificially magnifying the DE for small strains but also meant material behaviour at small strains would be clouded by the competing behaviours of differing loadcases condensed onto only 8-16 coefficients out of the 1728 available.

To alleviate these issues, the combined data set was characterised using two forms of strain space discretisation. Firstly, a uniform discretisation and then secondly, a two-way bias was applied to create a more highly discretised region in the central of the strain space. The new interval layout and increased discretisation afforded by the greater number of equations available, gave a final NRMSE of 2.78% as opposed to 8.89% when using the uniform spacing. The quality of the fit is also demonstrated by feeding the coefficients back into Equation (2), as shown in Fig. 4.

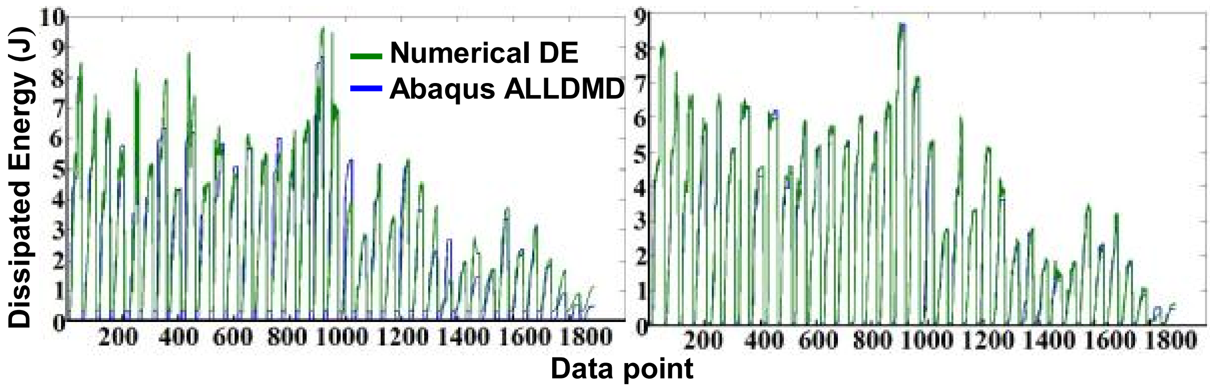

Fig. 4. DE predicted at all the data points. Left: Equal strain space discretisation. Right: Two-way bias discretisation.

Despite slight non-zero DE at small strains, Fig. 4 demonstrates that concatenating the four layup data sets produced DED coefficients that effectively recreated the DE seen across 37 individual synthetic loadcases with varying degrees of non-linearity.

Using the characterisation results from the combined data sets, a number of test cases were run in order to confirm that the fundamental DE parameter was performing as expected as the future evaluation of stiffness degradation was to be based upon DE. These tests included single element and characterisation specimens, where the quality of DE prediction was compared to the damage energy parameter ALLDMD, calculated by Abaqus.

Implementation into FEA

To be able to analyse the development of damage within notched CFRP coupons it was necessary to incorporate the DED coefficients and strain space data into an FE package and assess whether the methodology could capture behaviour for differing geometries and layups. The solver in Abaqus is able to interface with external files and execute commands through the use of user subroutines written in the Fortran environment. To this effect, subroutines were coded to fetch the characterisation constants and strain space information and subsequently calculate the energy dissipated by damage during an analysis. The constants and the strain space details are mutually dependent – the coefficients are of no use without details of the strain space used to characterise them. The flow chart in Fig. 5 illustrates the two subroutines most commonly used: a "passive" version which only calculates and provides DE as an output, and an "active" version that calculates DE and degrades the material properties of the model.

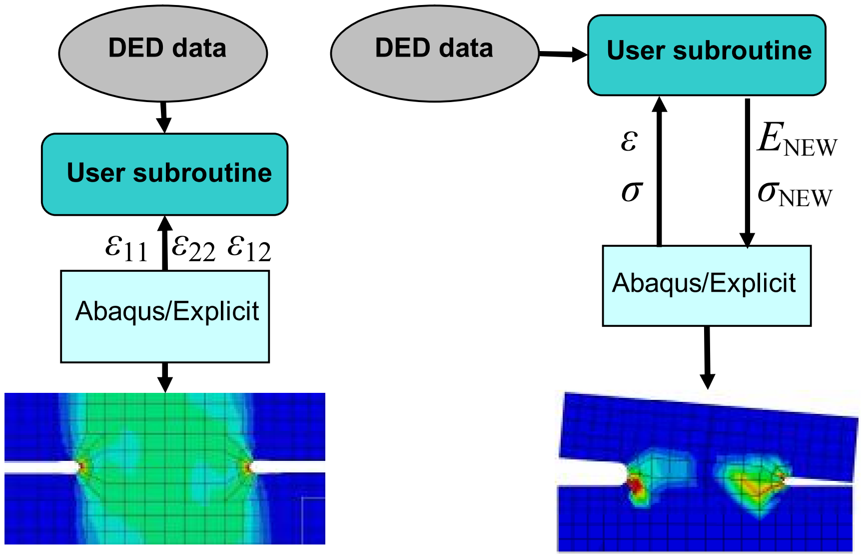

Fig. 5. Left: Passive DE calculation subroutine. Right: Active subroutine with calculation of DE and subsequent material softening.

Once an analysis has been initialised, the process of calculating the DE was exactly the same as the process used in characterising the DED constants, albeit now the unknown was the DE not the DED coefficients. When called by the solver, the subroutine first requested the strains at the material point for which it was being called. Then, in the process described previously, the position of the strain was located within the strain space whereupon it was then normalised and interpolated. These interpolated values were then multiplied by the DED coefficients on the boundaries of the respective strain space coefficients, and the volume of the element from which the material point came. This provided the energy dissipated by the current level of strain in a particular element. The total DE for the model was found by summing the energies from all elements.

Single Elements and Characterisation Specimens

For initial runs, the methodology was tested using pure in-plane loading cases, i.e. pure tension, shear and in-plane rotation (in the plane of the laminate). All the models assessed using the DED methodology were solved using the Hashin damage criteria so as to feed strains consistent with in-situ damage into the subroutine. The DE predicted by the subroutine was then compared with the Abaqus parameter ALLDMD calculated internally as part of the Abaqus damage model.

Fig. 6 provides a base comparison between the numerical DE and the Abaqus ALLDMD parameter for single element tests using ±15° and ±30° layups under uniaxial loading. In all cases the onset of damage was well predicted, especially in the slightly more diffuse damage modes seen in shear and rotational loading. The initial plateau of DE was over/under predicted in each case and although this is of little concern this discrepancy is useful in highlighting a drawback of using biased strain space segmentation; when using a limited data set, increased discretisation in the centre comes at the cost of the outer regions of the strain range become significantly coarse. Interpolation of a strain coordinate in these regions can become inaccurate and combined with the often sporadic non-linear strain response of the single element models this could affect the overall DE magnitude. Despite this, the accurate initiation responses and overall behaviour matched well with the Abaqus DE parameter and thus demonstrated the capabilities of the methodology and subroutine implementation.

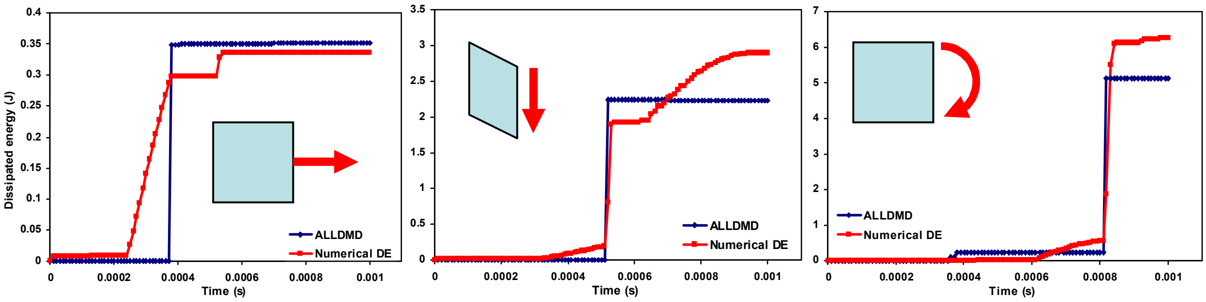

Fig. 6. Numerical DE compared with ALLDMD, ±15°layup with single axis loading. Left: Tension. Middle: Shear. Right: Rotation.

When applying the passive subroutine to more complicated specimens such as double notch coupons, it was expected that some of the idiosyncrasies peculiar to the single element models would disappear. Indeed this was the case as shown in Fig. 7, where comparisons of the numerical DE and ALLDMD are given for ±15°and ±30° layups. Of particular importance is the initiation and early progression of the DE as this corresponds to the beginning of the non-linear regime. In both cases the DED methodology captured the initiation very closely and also the sharp increases in DE. Abaqus predicted very similar DE behaviour for both layups, with plateaus occurring at approximately 2, 6 and 14 joules, and the numerical DE also reflected this behaviour.

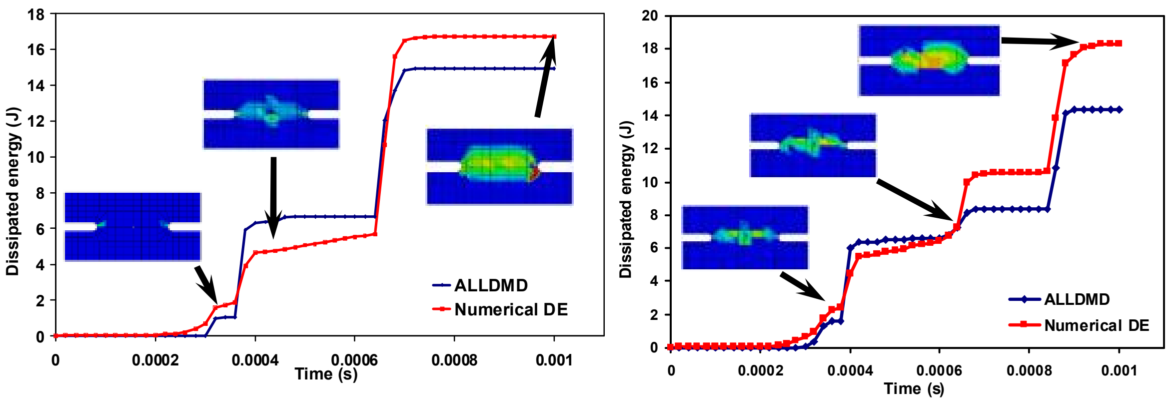

Fig. 7. Comparison numerical DE and ALLDMD. Left: ±15° layup. Right: ±30° layup.

Interestingly, though the increases in DE were well captured, the numerical method tended to plateau either earlier or later than the ALLDMD parameter. The initial rises in DE signalled the beginning of the non-linear regime and the associated large changes in strain. Larger strains were not characterised well due to the discretisation scheme explained previously. However, this is a numerical issue and readily improved by making use of additional data. For the purposes of the current constitutive modelling process, the initiation and formation of DE for the first failure event was well captured.

Dissipated Energy Based Constitutive Modelling

To link the DE development with a constitutive model, one method is to introduce a damage parameter. This parameter can then represent the degradation in material stiffness for a given value of dissipated energy due to mechanical strain. Preliminary work employed a single damage parameter which was calculated from the difference between the total work energy imparted to the model, *W*, and the energy dissipated. This damage factor, *d*, was applied to each component of the orthotropic stiffness matrix as shown in Equation (3) and (4).

$$W - DE = (1-d)W; \qquad d = 1 - \frac{W - DE}{W} \tag{3}$$

$$\begin{bmatrix} (1-d)C_{11} & (1-d)C_{12} & (1-d)C_{13} & 0 \\ & (1-d)C_{22} & (1-d)C_{23} & 0 \\ & & (1-d)C_{33} & 0 \\ sym & & & (1-d)C_{44} \end{bmatrix} \tag{4}$$

This so-called homogenous damage model was advantageous in its simplicity and allowed for confirmation of the linking process between the DE calculated through the subroutine and the updated stresses and energies passed back to Abaqus. This model was applied to a number of double notch characterisation specimens of various layups. The result of applying this model to the ±15° layup is given in Fig. 8; the sharp increase in the damage factor at 0.0001s adjacent to the notch area coincides with the initiation of energy dissipation, and as the damage energy increases so does the damage factor.

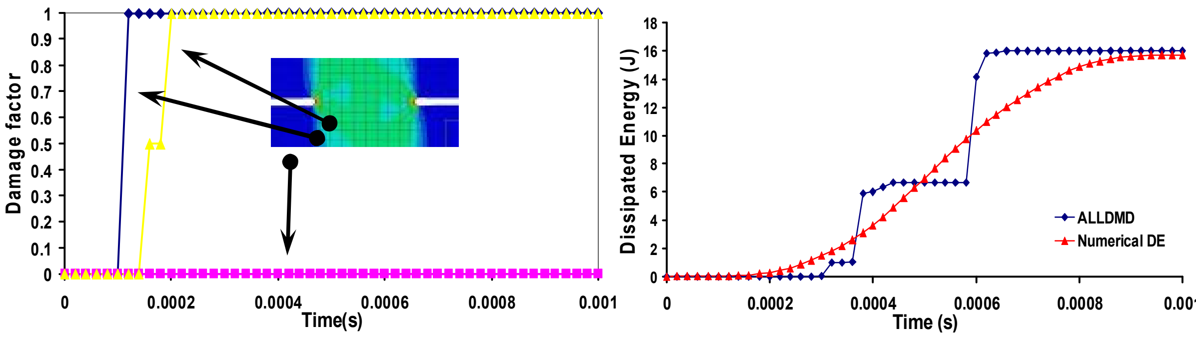

Fig. 8. Analysis of ±15° layup with DED constitutive model.
Left: Damage factor at different locations. Right: Numerical DE and ALLDMD.

Even though the behaviour of the energy being dissipated was altered, the initiation and progression of DE were reasonably well captured for the analysis and the overall magnitude by the end of the analysis was very close to that predicted by Abaqus. The blanket reduction of all the stiffness components produced a very gentle failure response – since the DE is calculated from strain, the resultant DE lacked the characteristic sharp increases and plateaus. Nevertheless, this exercise has proven that a relationship can be forged between DE and material stiffness. Despite the idealistic reduction of all stiffness components, the DE was realistic and maintained a similar behavioural trend – since the DE and stiffness are mutually affected by one another, improvements in the constitutive model will improve both.

Conclusion

A relationship between the DE and stiffness degradation has been established and modelling this stiffness loss has impacted DE predictions and the subsequent material response. Both the passive initiation and progression of DE was assessed as well as the active use of a simple constitutive model linking the DE and degradation in material stiffness. The basic material response over a range of different load combinations was compared with the Abaqus Hashin criteria, which was also used to characterise the DED data. Despite the simplistic homogenous damage model tested, the resulting realistic DE appears to support the postulation that DE can be a measure and a controlling parameter in the response of a material.

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